GEOMETRICAL SLIPPING IN ROLAMITE TYPE MECHANISMS

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Annotation

In the article is reviewed one variant of slipping, existing in the precision roller – band mechanisms (PRBM) such as rolamite type mechanism (RTM), i.e. geometric slipping, the theoretical investigation is given.

The rollers are wrapped by a flexible band in the mechanisms and contacted among themselves only through a flexible band, the ends of which one fasten to the directing planes of RTM.

In PRBM there is geometric sliding the value of which is influenced by thickness of flexible band. The compensation of geometric slipping may be achieved by introducing additional band covering rollers, direction of which is opposite to that of the main band.

Keywords: Rolamite type mechanisms, geometrical slipping, roller, band

Introduction

Donald F. Wilkes had invented the precision roller-band mechanism, called Rolamite (roller + mite) type mechanism (RTM) in 1967 [Wilkes, D.F. 1967; Wilkes, D.F. 1968].

The RTM rollers are wrapped around by flexible band with high tension in big angle (usually >180°); they are contacting through a flexible band which is attached with stretch by their own ends of two direct surfaces.

Articles [Wilkes, D.F. 1967; Wilkes, D.F. 1968; Cadman, R.V. 1969] indicate that RTM is precision mechanism, the elements of which are moving without slipping with regard to one another. However authors of the article [4] indicate that rollers in RTM slip under some determined parameters of the mechanism, but they do not present theoretical motivation of it.

The band wraps around all rollers (such as RTM or scroller [see *http://scroller-wheel.com*]) in precision roller-band mechanisms (PRBM). Thus possible geometrical slipping is stipulated by the presence of flexible transmission element (a band with finish thickness).

Geometrical slipping**,** that is relative displacement of touching spots in friction mechanisms, depends on the form of interacting bodies in the zone of their contact.

The purpose of the article is to find out how the geometrical slipping can be compensated in precision roller band mechanisms.

Theoretical research of geometrical slipping in roller – band mechanisms

Let's research a component characteristic to rolamite mechanisms and that of a scroller, which consists of two cylindrical rollers and a flexible band which is wrapped around them from the opposite sides (Fig. 1).

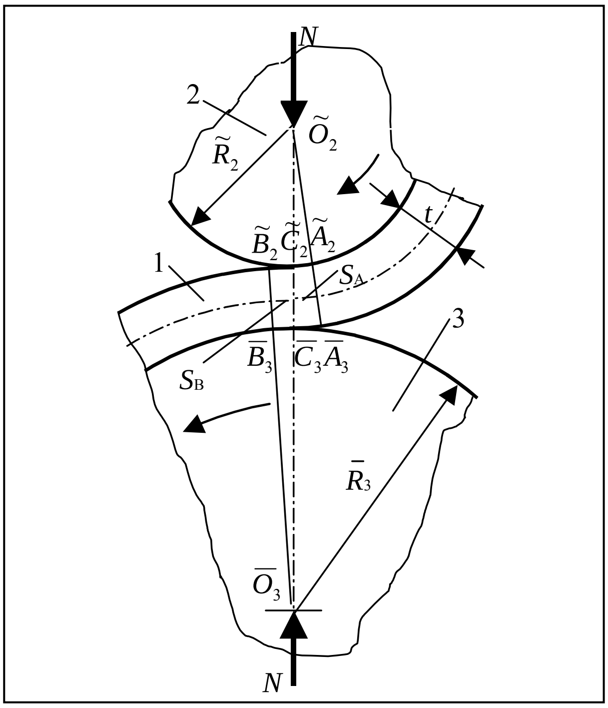

Fig. 1 Schematics of PRBM mechanism discussed to determine geometrical slipping of the elements

It is assumed that the band with the rollers, affected by external force, form a roller mechanism with very tight geometry; elements of the mechanism contact in the line of the centers of the rollers. The contact load of the rollers and the bands affect kinematics of the mechanisms discussed, because movement is performed in the points of contact transfer.

Let the band *1* move from the "feeding" roller *2*, which radius is $\tilde{R}_2$, on the "receiving" roller *3* which radius is $\bar{R}_3$. The "feeding" elements of the roller-band mechanism (the roller or the deflective plane) are marked as "~", and the "receiving" elements are marked as "–".

The section $\tilde{A}_2\ \bar{A}_3$ moves to position $\tilde{B}_2\ \bar{B}_3$ over their contact zone and becomes a natural extension of the lines $\tilde{O}_2\tilde{A}_2$ and $\bar{O}_3\bar{B}_3$.

Let's exam the movement of band *1* in contact zone of elements *2* and *3*.

The distance of the sections from the centre line $\tilde{O}_2\bar{O}_3$ to the border of the contact zone is marked $S_A + S_B$, thickness of the band *1* as t . Value of expression $S_A + S_B$ depends on material flexibility of the contacting elements *2-1-3*, the radius $\tilde{R}_2$ and $\bar{R}_3$ of the elements *2* and *3* curvature, and value of normal load *N*. Moreover, compression deformations occur when an external load influences the contact zone of the elements, and their resultant force passes through the rolling axis of the rollers. Redistribution of deformations occurs in the contact points when the mechanism loaded with force *N* is rolling. As the result, the point, where the resultant force operates, moves towards rolling side by some distance *k*. The reason for such redistribution of deformations is the elastic ridges on the bodies' surface. Rolling damping forces performs work, used for their formation.

Then according to Fig. 1 it is assumed that

$$S_A = S_0 + k\ ;\ S_B = S_0 - k\ ;\ S_A + S_B = 2\,S_0\ , \qquad (1)$$

k - coefficient of rolling damping,
$2\,S_0$ - contact's width.

It is assumed that

$$\min\,(\tilde{R}_2\,;\bar{R}_3) \gg t\ ;\quad \min\,(\tilde{R}_2\,;\bar{R}_3) \gg S_0 > k. \qquad (2)$$

Satisfactory reliable values $\tilde{B}_2\tilde{C}_2$, $\tilde{C}_2\tilde{A}_2$, $\bar{B}_3\bar{C}_3$, $\bar{C}_3\bar{A}_3$, displacement sector of each roller *2* and *3* in the contact zone, were found from the quadrangle $\tilde{A}_2\tilde{B}_2\bar{B}_3\bar{A}_3$ after evaluating smallness row of the accessed values:

$$\tilde{B}_2\tilde{C}_2 = S_B \frac{\overline{R}_3 + t}{\overline{R}_3 + \frac{t}{2}}; \quad \tilde{C}_2\tilde{A}_2 = S_A \frac{\tilde{R}_2}{\tilde{R}_2 + \frac{t}{2}};$$

$$\overline{B}_3\overline{C}_3 = S_B \frac{\overline{R}_3}{\overline{R}_3 + \frac{t}{2}}; \quad \overline{C}_3\overline{A}_3 = S_A \frac{\tilde{R}_2 + t}{\tilde{R}_2 + \frac{t}{2}}. \tag{3}$$

When section $\tilde{A}_2\overline{A}_3$ moves over into position $\tilde{B}_2\overline{B}_3$, the roller 2 surface will move over $\tilde{x}$, and roller *3* surface will move over $\overline{x}$:

$$\tilde{x} = \tilde{B}_2\tilde{C}_2 + \tilde{C}_2\tilde{A}_2 = \frac{(S_0 - k)(\overline{R}_3 + t)}{\overline{R}_3 + \frac{t}{2}} + \frac{(S_0 + k)\tilde{R}_2}{\tilde{R}_2 + \frac{t}{2}};$$

$$\overline{x} = \overline{B}_3\overline{C}_3 + \overline{C}_3\overline{A}_3 = \frac{(S_0 - k)\overline{R}_3}{\overline{R}_3 + \frac{t}{2}} + \frac{(S_0 + k)(\tilde{R}_2 + t)}{\tilde{R}_2 + \frac{t}{2}}. \tag{4}$$

The flexible band *1* will move over:

$$2S_0 = \frac{\tilde{x} + \overline{x}}{2}. \tag{5}$$

The band *1* is moved over infinitively small value ds and the rollers *2* and *3* are moved over d$\tilde{x}$ and d$\overline{x}$ respectively.

Considering that:

$$d\tilde{x} = \tilde{x}\frac{ds}{2S_0}; \quad d\overline{x} = \overline{x}\frac{ds}{2S_0}. \tag{6}$$

the movements of the rollers *2* and *3* (d$\tilde{x}$ and d$\overline{x}$) conform to the movement of the flexible band *1* (ds) when

$$\frac{t}{R} \ll 1: \tag{7}$$

can be expressed like:

$$d\tilde{x} = \left[\frac{(S_0 - k)(\overline{R}_3 + t)}{\overline{R}_3 + \frac{t}{2}} + \frac{(S_0 + k)\tilde{R}_2}{\tilde{R}_2 + \frac{t}{2}}\right]\frac{ds}{2S_0} = \frac{1}{2}\left[\frac{(1 - \frac{k}{S_0})(1 + \frac{t}{\overline{R}_3})}{1 + \frac{t}{2\overline{R}_3}} + \frac{1 + \frac{k}{S_0}}{1 + \frac{t}{2\tilde{R}_2}}\right]ds. \tag{8}$$

Accordingly, Equation (7) can be written that

$$\frac{t}{2\overline{R}_3} \ll 1; \qquad \frac{t}{2\tilde{R}_2} \ll 1. \tag{9}$$

After a variation of these small values the following equation is derived:

$$d\tilde{x} = \left[1 + \frac{t}{4}\left(\frac{1}{\overline{R}_3} - \frac{1}{\tilde{R}_2} - \frac{k}{\overline{R}_3 S_0} - \frac{k}{\tilde{R}_2 S_0}\right)\right]ds. \tag{10}$$

Disregarding the squares of the small values it is received:

$$d\tilde{x} \cong \left[1 + \frac{t}{4}\left(\frac{1 - \frac{k}{S_0}}{\overline{R}_3} - \frac{1 + \frac{k}{S_0}}{\tilde{R}_2}\right)\right]ds. \tag{11}$$

Analogically

$$d\overline{x} \cong \left[1 - \frac{t}{4}\left(\frac{1 - \frac{k}{S_0}}{\overline{R}_3} - \frac{1 + \frac{k}{S_0}}{\tilde{R}_2}\right)\right]ds. \tag{12}$$

It can be marked

$$m = \frac{k}{S_0} \text{ - coefficient of contact} \tag{13}$$

and changes with conversion of signs are performed:

$$\mathrm{d}\,\tilde{x} = \left[1 - \frac{t}{4}\left(\frac{1+m}{\tilde{R}_2} - \frac{1-m}{\bar{R}_3}\right)\right]\mathrm{d}s; \tag{14}$$

$$\mathrm{d}\,\bar{x} = \left[1 + \frac{t}{4}\left(\frac{1+m}{\tilde{R}_2} - \frac{1-m}{\bar{R}_3}\right)\right]\mathrm{d}s. \tag{15}$$

If it is assumed that

$$\delta = \frac{t}{4}\left(\frac{1+m}{\tilde{R}_2} - \frac{1-m}{\bar{R}_3}\right), \tag{16}$$

then measurable displacement of respective rollers in the contact zone is

$$\mathrm{d}\,\tilde{x} = (1-\delta)\,\mathrm{d}s; \;\; \mathrm{d}\,\bar{x} = (1+\delta)\,\mathrm{d}s, \tag{17}$$

where δ means kinematical coefficient of slipping.

Equations (17) show that the rollers in relation to the band slips to the opposite direction (Fig. 1). Slipping of both rollers, if viewed from an absolute point, is single-sided, but movement of the band "feeding" roller *2* (with $\tilde{R}_2$) becomes slower, because slipping is oriented to direction opposite to rolling, meanwhile the "receiving" roller *3* (with $\bar{R}_3$) – quickens. When moving is opposite, the roller "feeding" band becomes "receiving", and the "receiving" – "feeding", their slipping direction doesn't change. It can be concluded that there exists kinematically irreversible geometrical slipping of the elements of the PRBM mechanism discussed, when radii of the rollers are freely chosen.

The kinematical coefficient of slipping δ is different for opposite displacements of the rollers, because radii values of the band "feeding" and "receiving" rollers in the equation (16) exchange. It can be seen (16) that δ is determined by geometrical characteristics of the mechanism discussed, but thickness *t* of the flexible band essentially influences the geometrical slipping value of its elements

The equation (16) for identical rollers would be:

$$\delta = \frac{mt}{2R}. \tag{18}$$

Compensation of geometrical slipping in roller – band mechanisms

Using theoretical conclusions about causes of occurring of geometrical slipping of PRBM elements, let us examine straight-line reversionary movement of a roller wrapped with a flexible band and rolling over a plane without slipping in the zone of contiguity (Fig. 2)

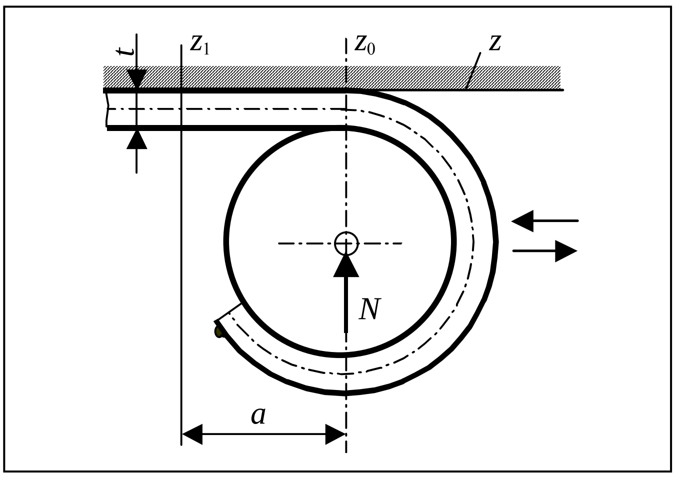

Fig. 2 Schematic diagram of a roller – band mechanism used to examine geometrical slipping

The roller with the band displaces from position z_1 into position z_0 and "provides" the band. The flat joint of the mechanism will become "receiving" band and according to the equations (17), when dz > 0, we may write equation of the roller displacement depending on displacement of the band

$$dz = (1 + \delta_1)ds, \quad (19)$$

z – longitudinal displacement of the roller;
s – displacement of the band;
δ_1 – kinematical coefficient of slipping (rightward displacement)

When dz < 0, the flat joint will become the "feeding" band and the dependence will look like this:

$$dz = (1 - \delta_2)\, ds', \quad (20)$$

δ_2 – kinematical coefficient of slipping (leftward displacement)

Taking into account that δ_1 and δ_2 are small values, dependences of displacement of the band in each case will be:

$$ds = (1 - \delta_1)dz; \qquad ds' = (1 + \delta_2)dz. \quad (21)$$

After integrating:

$$s_1 - s_0 = (1 - \delta_1)(z_1 - z_0); \quad (22)$$

$$s_0' - s_1 = (1 + \delta_2)(z_0 - z_1). \quad (23)$$

After completion of the equations (22) and (23) it is found:

$$s_0' - s = (z_1 - z_0)(-\delta_1 - \delta_2); \quad (24)$$

$$\Delta s = -a(\delta_1 + \delta_2), \quad (25)$$

a – amplitude of displacement of the roller;
Δs – magnitude of displacement of the band in one cycle of roller rolling;
"–"indicates that the direction of displacement is opposite to the direction of z

We find values of δ_1 ir δ_2 in the equation (16), taking into account that radius of one roller $R = \infty$ (plane)

$$\delta_1 = \frac{t}{4}\cdot\frac{1+m}{R}; \qquad \delta_2 = -\frac{t}{4}\cdot\frac{1-m}{R}; \quad (26)$$

$$\delta_1 + \delta_2 = \frac{tm}{2R}; \qquad \Delta s = -\frac{atm}{2R}. \quad (27)$$

The irreversibility of the geometrical slipping of PRBM elements towards the direction of the movement and to compensate such slipping we offer to implement an additional flexible joint in the mechanism – a band, wrapping the roller from the opposite side. That would allow getting constant transfer dependences between angular and linear displacements of the elements and thus compensating geometrical slipping between the elements of the mechanism.

The schematics of a mechanism in which geometrical slipping between the elements is compensated is given in Fig. 3. The roller wrapped with two bands from the opposite sides is moving on the plane z. The bands are stretched with force T, and the roller is loaded with force N and that ensures tight contact of the contiguous elements

Taking into account that the roller is wrapped with two bands, we have a possibility to examine the contiguity kinematical pair "roller – plane" as "band-feeding" and "band-receiving" at the same time.

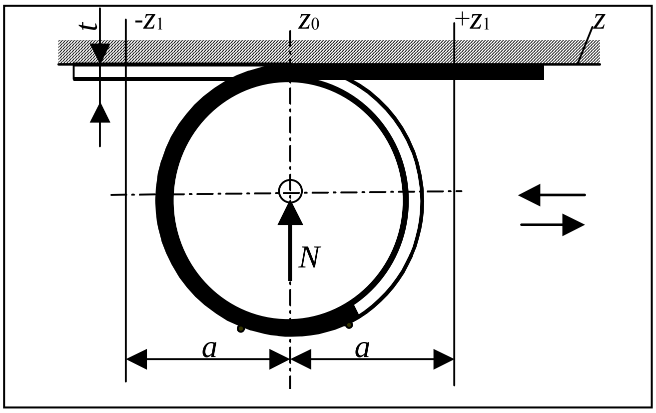

Fig. 3 Compensation of geometrical slipping in roller – band mechanisms

According to the direction of the movement we receive such dependences:

$$\Delta s_1 = -a(\delta_1 + \delta_2); \quad (28)$$

$$\Delta s_2 = a(\delta_1 + \delta_2). \quad (29)$$

The total result of dependences (28) and (29) will be equal zero, i.e.

$$2\Delta s = \Delta s_1 + \Delta s_2 = 0,$$

which proves that it is possible to compensate for the geometrical slipping between the elements by wrapping the roller with two bands of opposite direction

Dependences (28) and (29) may differ not by their sign only, but also by values of kinematical coefficients, according to different conditions of forward and backward movement. In this case value of deviation may be different from zero, they are equal to absolute magnitude, and opposite by their sign.

The original roller – band mechanisms with compensation of the geometrical slipping were invented [SU invention No 1516348. Int. Cl.[4] B25J 15/02; B25J 11/00; SU invention No 1566851. Int. Cl.[4] F16H 19/06; H02N 2/00] and designed on the grounds of the conclusions of the work about existing geometrical slipping between the elements of the roller - band mechanisms, and theoretical study about a possibility to compensate such slipping.

Conclusions

Theoretical research was proved:

1. There exists kinematically irreversible geometrical slipping of the elements of PRBM. Its magnitude is influenced by thickness of the flexible band.

2. Geometrical slipping between the elements of PRBM may be compensated, if an additional flexible band, wrapping the rollers from the opposite side, complements the design.

3. In spite of constructional, technological and operational errors of RTM, the main influence on the kinematic precision is made by geometrical slipping of the elements because of imperfection of the structural links.

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GEOMETRINIS SLYDIMAS ROLAMAITO TIPO MECHANIZMUOSE

Straipsnyje išnagrinėtas vienas slydimo, egzistuojančio tiksliuose juostiniuose-ritininiuose mechanizmuose (TJRM), tokiuose kaip rolamaito tipo mechanizmai (RTM), variantas, t.y. geometrinis slydimas. Teoriniai tyrimai pateikti.

Ritinėliai mechanizmuose yra gaubiami lanksčia juosta ir kontaktuoja tarpusavyje tik per lanksčią juostą, kurios galai pritvirtinti prie kreipiančiųjų RTM paviršių.

TJRM egzistuojančio geometrinio slydimo reikšmė priklauso nuo lanksčios juostos storio. Geometrinį slydimą galima kompensuoti įvedant papildomą ritinėlius gaubiančią juostą kryptimi priešinga pagrindinei juostai.